Distributed Information Processing

2nd Lecture

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Outline

- Clock
  - Logical Clocks & Ordering of Events
  - Vector Clocks
- Q&A

Partial Ordering of Events

- Assumptions
  - System Consists of N Processes:

$$p_i \quad (i = 1, \ldots, N)$$

  - System Is Composed of a Collection of Events
    - Examples of events
      - Execution of a subprogram
      - Execution of a single machine instruction
      - Sending or receiving a message

$$history(p_i) = h_i = \left\langle e_i^k \,\middle|\, k = 1, \ldots \right\rangle \quad (i = 1, \ldots, N)$$

Partial Ordering of Events [Lamport78]

- "Happened Before" Relation ($\rightarrow$)

(1) $k < l \Rightarrow (if) \quad e_i^k \rightarrow e_i^l \quad (i = 1, \ldots, N)$

(2) $e_i = send(m), e_j = receive(m)$ *for Message m*

$\Rightarrow \quad e_i \rightarrow e_j$

(3) $e \rightarrow e' \wedge e' \rightarrow e'' \Rightarrow e \rightarrow e''$ — Transitive

* $e \not\rightarrow e$ — Irreflexive

- Concurrent Relation ($||$)

$e \not\rightarrow e* \wedge e* \not\rightarrow e \Leftrightarrow e || e*$

Logical Clocks [Lamport78]

- Ways of Assigning Numbers to Events

$$LC_i(e), \text{ where } e \text{ is an event in process } p_i$$

- Clock (Correctness) Condition

$$e \rightarrow e^* \quad \Rightarrow \quad LC(e) < LC(e^*)$$

  - Satisfied if the following two conditions hold:

$$(1)\ k < l \quad \Rightarrow \quad LC(e_i^k) < LC(e_i^l) \quad (i = 1, \ldots, N)$$

$$(2)\ e_i = send(m),\ e_j = receive(m) \quad \text{for Message } m$$

$$\Rightarrow \quad LC(e_i) < LC(e_j)$$

Logical Clocks (Cont'd) [Lamport78]

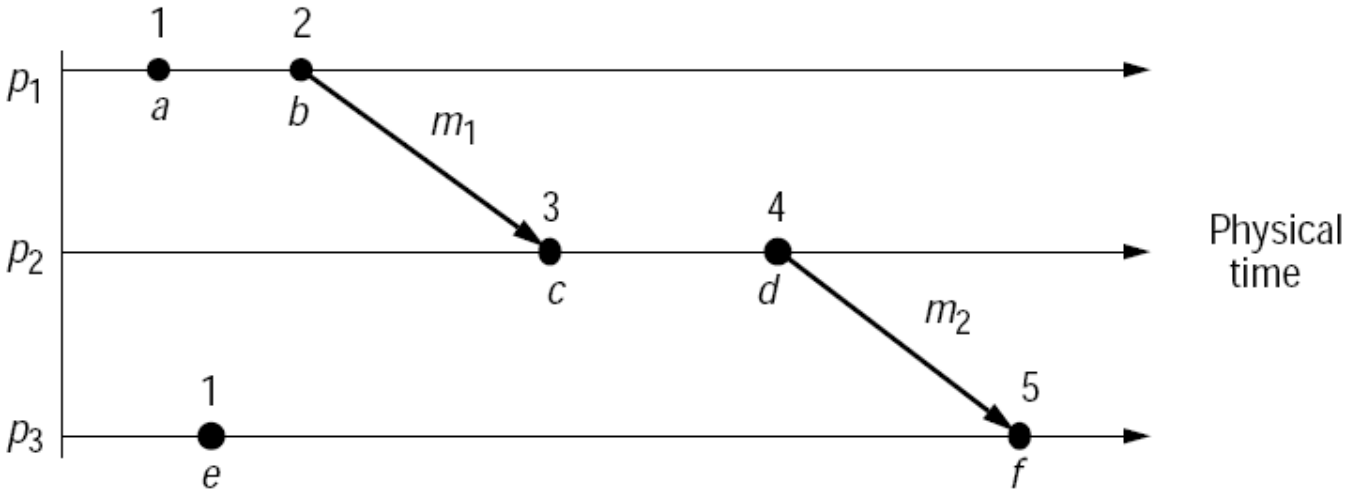
- Implementation Rules

(1) $LC(e_i^{k+1}) = LC(e_i^k) + 1$ *for Successive Events* e_i^k, e_i^{k+1}

$(i = 1, ..., N)$

(2) $e_i = send(m), e_j^l = receive(m)$ *for Message* m

$$\Rightarrow \quad t = LC(e_i) \in m, LC(e_j^l) = \max\{LC(e_j^{l-1}), t\} + 1$$



Ordering the Events Totally [Lamport78]

- Total Ordering ($\Rightarrow_{t.o.}$)

Arbitrary

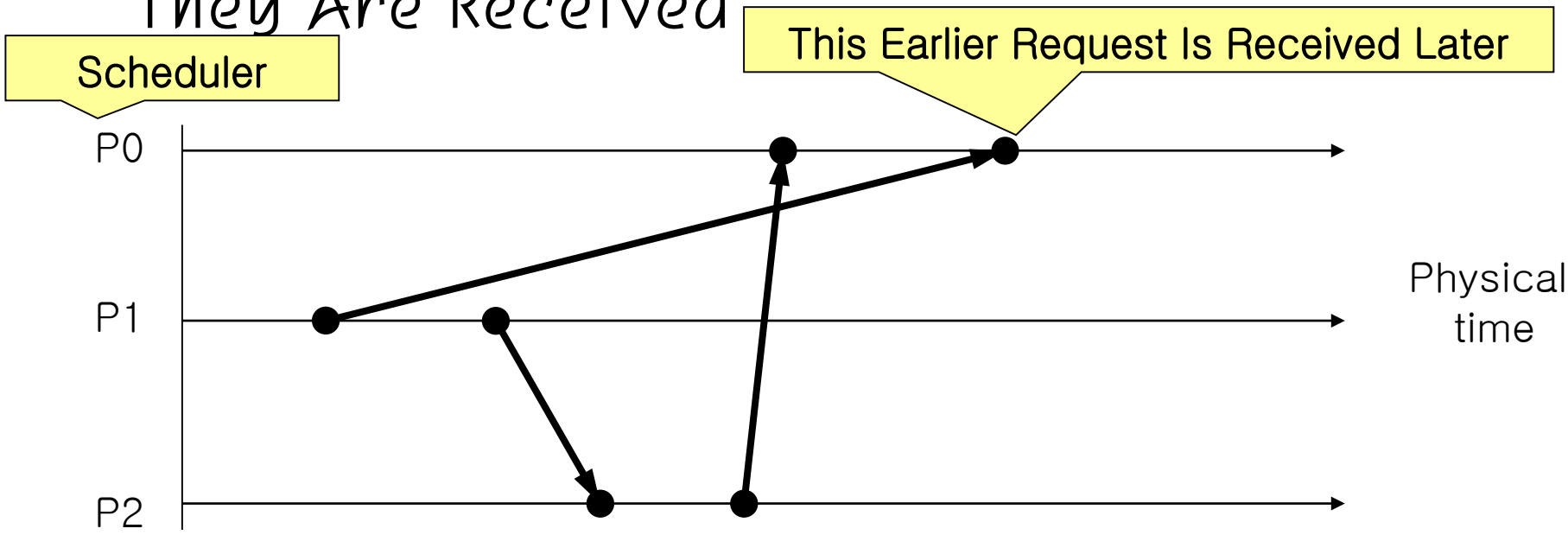
$$e \Rightarrow_{t.o.} e^* \quad iff \quad LC_i(e) < LC_j(e^*) \vee \{LC_i(e) = LC_j(e^*) \wedge i < j\}$$

$$e \rightarrow e^* \quad \Rightarrow \quad e \Rightarrow_{t.o.} e^*$$

  - Use Case: Mutual Exclusion Problem
    - Processes sharing a single resource
    - (1) A process using it must release it before the following use
    - (2) The processes must use it in their request order
    - (3) If it is eventually released, every request is eventually granted

Ordering the Events Totally (Cont'd) [Lamport78]

- Nontrivial Scheduling Problem
  - The Requests Cannot Be Granted in the Order They Are Received



Ordering the Events Totally (Cont'd)

(1) A process using it must release it before the following use
(2) The processes must use it in their request order
(3) If it is eventually released, every request is eventually granted

- Nontrivia
  - Assumptions for an Algorithm
    - In-order message delivery
    - Each process' request queue
      - Initially containing a request with T0:P0

Smallest Time Value: Scheduler ID

  - Algorithm

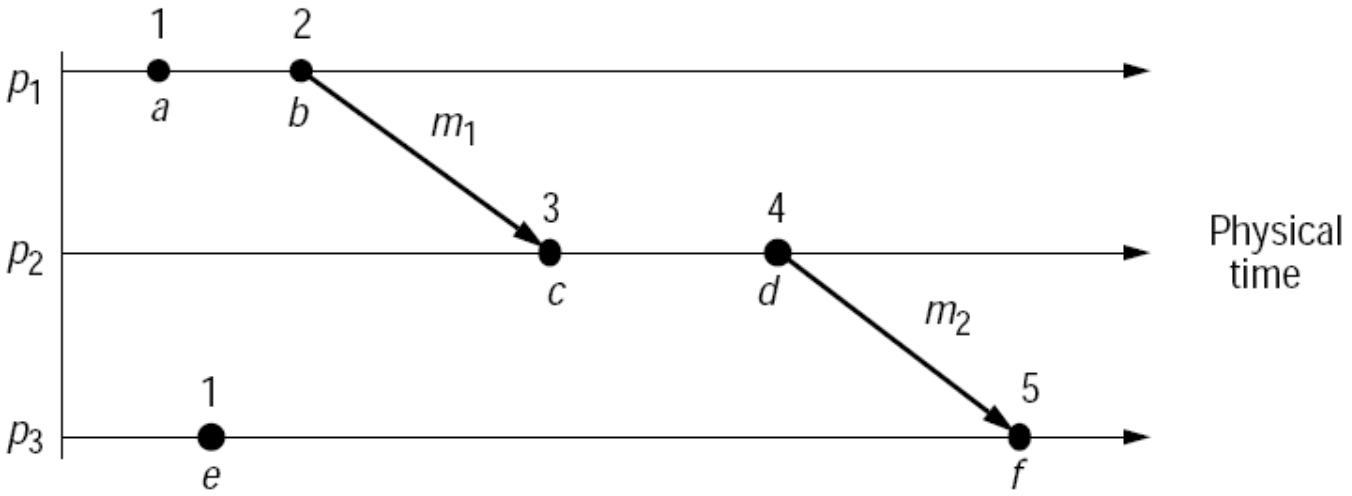
Send Time

    - Pi sends a Tm:Pi Req to all others and puts it in the queue
      - Each receiver puts it in the queue and sends a timestamped Ack to the requestor
    - When releasing the resource, Pi removes the Req and sends a timestamped Rel to all others
      - Each receiver removes Pi's Req from its queue
    - The Tm:Pi Req is granted if it is the first in $\Rightarrow$, and Pi has received a message from all others timestamped later than Tm

Vector Clocks

- Shortcomings of Lamport's Logical Clock

$$LC_i(e) < LC_j(e^*) \quad \not\Rightarrow \quad e \rightarrow e^*$$



Vector Clocks (Cont'd)

- Implementation Rules

(1) $VC_i[j] = 0 \; (i, j = 1, ..., N)$

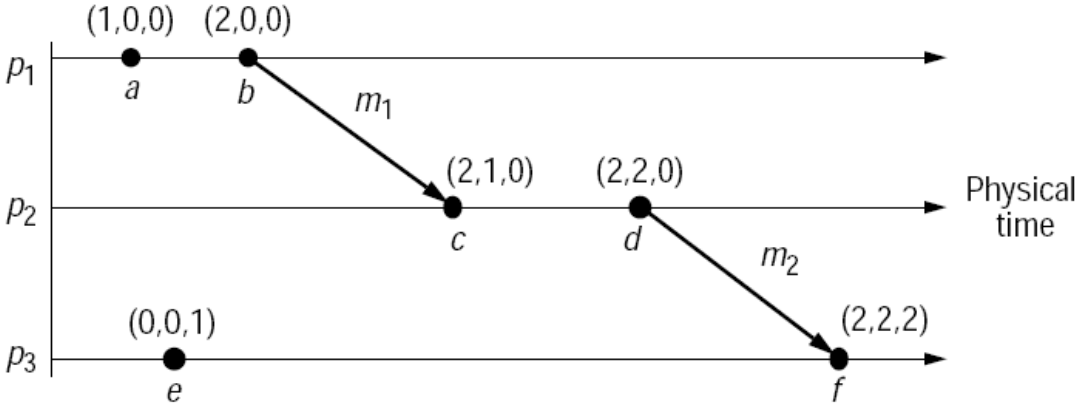
(2) $VC_i(e_i^{k+1})[i] = VC_i(e_i^k)[i] + 1$ *for Successive Events* e_i^k, e_i^{k+1}

$(i = 1, ..., N)$

(3) $e_i = send(m), e_j^l = receive(m)$ *for Message* m

$$\Rightarrow \quad t = VC_i(e_i) \in m, VC_j(e_j^l) = \max\{VC_j(e_j^{l-1}), t\} +_j 1$$

Increment the j-th Elt



Vector Clocks (Cont'd)

- Vector-Timestamp Comparisons

$$VC = VC^* \; iff \; VC[j] = VC^*[j] \; (j = 1, ..., N)$$
$$VC \leq VC^* \; iff \; VC[j] \leq VC^*[j] \; (j = 1, ..., N)$$
$$VC < VC^* \; iff \; VC \leq VC^* \wedge VC \neq VC^*$$

- Properties

$$e \rightarrow e^* \; iff \; VC(e) < VC(e^*)$$

Strong Clock Condition

  - Hints for the Proof

$$e \| e^* \Rightarrow \neg\{VC(e) \leq VC(e^*) \vee VC(e) \geq VC(e^*)\}$$
$$(Note\ That\ VC_i[j] \leq VC_j[j])$$

Vector Clocks

- Properties

$$e_i \rightarrow e_j \ iff \ VC_i[i] \leq VC_j[i], \ where \ i \neq j$$

Simple Clock Condition

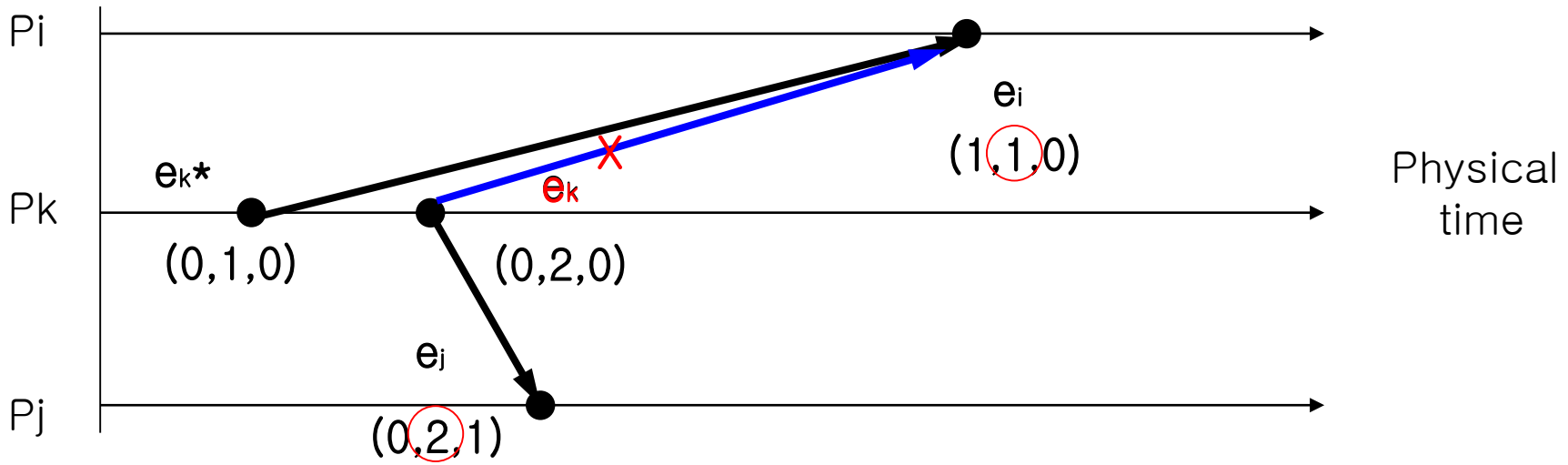
  - Hint for the Proof

$$(Note\ That\ VC_i[j] \leq VC_j[j])$$

$$\exists k (\neq j)\ s.t.\ VC_i[k] < VC_j[k]$$

Weak Gap-Detection

$$\Rightarrow \quad \exists e_k\ s.t. (e_k \not\rightarrow e_i) \wedge (e_k \rightarrow e_j)$$



Vector Clocks (Cont'd)

- Properties

Weak Gap Detection

$$\exists k(\neq j)\ s.t.\ VC_i(e_i)[k] < VC_j(e_j)[k]$$

$$\Rightarrow \quad \exists e_k(\neq e_i)\ s.t.\ (e_k \not\rightarrow e_i) \wedge (e_k \rightarrow e_j)$$

$$Suppose\ \ That\ i = k,$$

$$\exists k(\neq j)\ s.t.\ VC_k(e_k{*})[k] < VC_j(e_j)[k]$$

$$\Rightarrow \quad \exists e_k(\neq e_k{*})\ s.t.\ (e_k \not\rightarrow e_k{*}) \wedge (e_k \rightarrow e_j)$$

$$iff \quad \exists e_k(\neq e_k{*})\ s.t.\ e_k{*} \rightarrow e_k \rightarrow e_j$$

$$\exists e_k(\neq e_k{*} \wedge k \neq j)\ s.t.\ e_k{*} \rightarrow e_k \rightarrow e_j$$

$$\Rightarrow \quad VC_k(e_k{*})[k] < VC_j(e_j)[k]$$

Vector Clocks (Cont'd)

- Properties

$$\forall (k \neq j),\ VC_k(e_k{*})[k] \geq VC_j(e_j)[k]$$

$$\Rightarrow \neg\{\exists e_k (\neq e_k{*})\}\ s.t.\ e_k{*} \rightarrow e_k \rightarrow e_j$$

Contraposition

  - If the Monitoring Process Keeps the Value of the k-th Elt of VC from Pk, It Can Decide Whether There Is No Event That Happened before a Given Event